Translating between verbal and algebraic expressions for the Retail Donation Program (RDP) – J. Warfel, D. Wilson

Purpose

The ability to translate between equivalent verbal and algebraic expressions is a fundamental skill of high school algebra. This lesson teaches and practices that skill in the context of a logistics system common at food banks, an interesting real-world application.

Overview

In this lesson, students will translate between verbal and algebraic expressions using variables that describe a logistics system common at food banks called the "Retail Donation Program" or "RDP." The lesson begins with a brief lecture that explains the system and defines the variables to be used. Then, students participate in a matching activity. Each student has a pair of paper strips, one with a verbal expression and the other with an algebraic expression, and must find the student who has the matching pair. After they find their matches, the students explain how the verbal and algebraic expressions are equivalent to justify their match. The lesson includes homework that uses the set of variables defined in class.

Student Outcomes

Students will be able to interpret algebraic statements that arise in a mathematical model.
Students will be able to represent verbal expressions and statements using algebraic variables.

Associated standards:

Common Core A-CED-2 (Create equations in two or more variables) and A-CED-3 (Represent constraints by equations or inequalities); also addresses Common Core standards for High School – Modeling.
Illinois Learning Standards for Mathematics: Goals 8.A.4b, 8.B.4a, 8.D.4
Texas Math TEKS: Algebra I 1.D, 3.A. 3.B; Math Models 1.B

Time

This lesson can be completed in one class period of 50 minutes, though this may not leave time for the presentation and discussion portion of the lesson. Alternatively, the lesson can be scheduled for a longer time period (such as 70 minutes) or over two class periods, with the lecture and matching activity in the first period and the presentation and discussion portion starting the second.

Level

High School: Algebra I, Algebra II, or Math Models

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Materials and Tools

Handout for each student (structured notes on one side, homework on the other; there is an annotated version for the teacher as well.)
Verbal and algebraic expression strips, cut and paired by number
Pieces of blank paper (approximately one for every two students) and something that can be used to attach the strips to that paper (such as glue or staples)

Preparation

The verbal and algebraic expression strips, in a separate document, must be printed (one-sided) and cut so that each student has a pair of strips with the same number. One of these strips will have a verbal expression or statement, and the other will have an algebraic expression or statement. Thirty pairs of strips are provided. If there are fewer than thirty students in the class, some of the strips should be discarded, but it is important to only discard them in matching pairs. Alternatively, some students could be given multiple pairs of strips. If there are an odd number of students in the class, some odd number of advanced students could be given multiple pairs of strips, or a pair of students could be assigned to share a pair of strips.
For clarification, see the Key, a separate document.

Prerequisites

Some of the variables used in this lesson have subscripts. If students are not familiar with subscripts, this should be explained before or during the lecture portion of the lesson.

Background

This lesson is based on a system called the "Retail Donation Program," which we abbreviate "RDP." This is a program used by food banks throughout the United States. In this program, the food bank sends out a truck to pick up donations of food from grocery stores (which we call "donors") and to deliver food to organizations that help hungry people, such as food pantries, shelters, and soup kitchens (which we collectively call "agencies").
We can model the RDP mathematically by assigning variables to important quantities in the problem, such as the capacity of the truck (how much food it can carry), the amount of food we put in the truck before it leaves the food bank's warehouse, how much food is given in donations, how much food is delivered at agencies, and what the demand is at the agencies (how much food they want for the people they serve). We can then use these variables to build a mathematical model of the system, which can be analyzed to find ways to help the system run more efficiently. However, in this lesson we will only consider how to use the variables we have defined to make algebraic expressions and statements.

Teaching Notes

This lesson has three parts: A short lecture, a matching activity in which each student uses the information from the lecture to match a pair of paper strips with another student's pair, and an optional discussion and presentation of the matched pairs.
Lecture:
Use the guided notes side of the worksheet to define the eight variables that will be used in the activity. We suggest first describing the RDP system with the diagram and writing the variables on the diagram, as demonstrated in the annotated handout, then writing more formal definitions of the variables in the table. Many of the variables have very similar definitions; therefore, after presenting formal definitions for W_1, R_1, and d_1, have students write formal definitions for W_2, R_2, and d_2 on their own, and use the opportunity to walk around the room and perform a check for understanding on every student. It is

crucial that most of the students understand the definitions of the eight variables well at this point, in order that they are able to complete the matching activity as a class.

After the variables have been defined, provide solutions and explanations for some or all of the six examples at the end of the handout. Alternatively, some or all of these could be given to the class to complete independently, and then discussed. We suggest that you do at least one example in each direction (from verbal to algebraic and from algebraic to verbal), and that you do at least one example that uses the variable for capacity (Q), since this vocabulary or concept may be unfamiliar to students.

Matching Activity:

Give each student a pair of slips with the same number (slips are numbered from 1 to 30). The numbers are randomly assigned and have no relationship to the information on the slip or to the difficulty of the problem; they are provided only to ensure that each student has a pair that exactly matches one other student's pair, and for ease of checking answers. For example, the student with pair number 1 should find the student with pair number 27, because algebraic statement 1 is described by verbal statement 27, while verbal statement 1 describes algebraic statement 27. The numbers for matches are listed in the Key, a separate document.

After each student has a pair of slips, instruct the students to walk around the room to find their match. That is, each student has an algebraic expression or statement, and is looking for a student who has a verbal expression or statement that matches the algebraic one, and vice-versa. When two students have found that their pairs match, they should attach their strips to a piece of paper and, on the paper, write an explanation of how they know their answer is correct. If possible, check that students are correct before they attach their strips, because if they are incorrect, there will be at least two other students in the room who cannot find their match. A detailed procedure for the matching activity (including a table that can be used to quickly determine if students' answers are correct) is included in the Key.

The pairs are constructed so that half of the pair is relatively easy while the other is more difficult. The intention is that the students will learn from writing the explanation of the more difficult half of the pair.

Discussion and Presentation (optional):

If time permits, after all matches have been found, have each pair present their pair and explain briefly how they knew that their pairs were a match. Then, have the class discuss how they could formulate algebraic expressions and statements of their own using these eight variables. Some points that could be emphasized while leading this class discussion:

- The distinction between algebraic expressions (which consist of variables and operands only) and algebraic statements (which include an equals sign or inequality symbol). In their verbal form, algebraic statements correspond to sentences (which make a statement about the system), while expressions only correspond to phrases describing a quantity of interest.
- It is possible for the same algebraic expression to correspond to several different verbal expressions; there are examples of this among the strips.
- Looking at specific algebraic statements, you could discuss why the logic of the verbal statement required a specific inequality symbol to be used, such as $\geq$ instead of $>$.

Homework:

The homework consists of problems in which verbal statements must be matched to relatively simple algebraic ones (similar to the matching activity in class), then several problems in which students must provide an algebraic formulation for a verbal prompt. All of the problems use the eight variables defined in class. Note that although correct solutions have been provided, they are not unique (due to commutativity).

Possible extensions to the homework:

- Have students generate additional algebraic expressions and statements and provide their verbal interpretation.
- Have students describe another system, define variables for that system, then use those variables to create expressions and statements.

Assessment

There are several opportunities to check for understanding while giving the lecture and during the matching activity. The explanations given by students about why their pairs of strips match can also be graded, as can the homework.

The annotated handout provides suggested answers to the homework. The matching strips key provides the numbers of the pairs that match one another, as well as a table that includes all of the problems used in the strips, the examples, and the homework.

Additional Information

"Retail Donation Program" is a generic name for the type of program described here. Food banks often have a different name for their version of the program, such as "Food Rescue Program" or "Food Recovery Program." To find out more about the version of the program that is operating in your area, you can contact your local food bank. To find your local food bank, search the directory at the website of Feeding America.

The version of RDP that we have presented includes deliveries to agencies. However, relatively few food banks make deliveries to agencies as a part of their RDP program – most just use the program to pick up donations[1]. There are several possible benefits to including agencies: food is delivered to agencies more quickly and frequently; a smaller truck is required to pick up the same amount of donations; and less food must be processed at the food bank warehouse. However, there are also challenges to including agencies because the amount of food donated by grocery stores is random and is unknown until the RDP driver picks up the food. The version we present includes agencies in order to resemble the model that is currently being used to research how agencies can be most efficiently included in RDP routes.

We would like to improve this lesson plan. If you use it, or it inspires to create a similar lesson, please contact us to let us know about your experience: joseph.warfel@u.northwestern.edu, lphscs@gmail.com

[1] Some food banks that currently include agencies in their RDP routes include the Greater Chicago Food Depository, the Northern Illinois Food Bank (in Geneva, Illinois), and Second Harvest Heartland (in St. Paul, Minnesota).