A Week of Astrophysics
The Lives and Deaths of Stars, Einstein's Gravity, and Ripples in Space

Michael Zevin (with GionMatthais Schelbert)

Purpose

To teach students about the lives of stars, the objects they evolve into, Einstein's gravity, and gravitational waves. Stars are the backbone of virtually all areas of astrophysics, from planet formation to galaxy evolution. The remnants that stars become, such as black holes and neutron stars, have recently also provided astronomers a new means for exploring the Universe. In this series of lessons, we dive deeper than the typical understanding of stellar lives and deaths and also get a taste of general relativity, gravitational waves, and data analysis techniques for how to find this elusive phenomenon.

Overview

Each day will center around a particular topic and hands-on demo.

Day 1 (Lives of stars, hydrostatic equilibrium, nuclear fusion) will use simple math to understand the energy output of the Sun and a balloon wrapped in aluminum foil demo as an analogy to hydrostatic equilibrium.
Day 2 (supernovae and stellar remnants) will teach why supernovae occur using the supernova "energy balls" demo, learn about what is left behind after supernovae, and do some simple math to learn about how extreme compact objects are.
Day 3 (general relativity) will teach about modern gravity and help students understand how this description of gravity gives us the gravitational force that we're familiar with using the spacetime table demo.
Day 4 (gravitational waves) will be mostly presentation and will discuss how gravitational waves come out of general relativity, highlight the recent discoveries, and discuss what these discoveries taught us about stellar remnants such as black holes and neutron stars.
Day 5 (gravitational wave detection/data analysis) will discuss interferometry (with a tabletop interferometer demo, if possible), how the LIGO detectors work, and data analysis techniques for detecting gravitational waves (matched-filtering demo).

Student Outcomes

- Students will be able to describe the concept of hydrostatic equilibrium, and understand the physical processes that balance stars
- Students will be able to understand why stars end their lives, and qualitatively how supernova explosions operate
- Students will be able to describe different compact objects such as white dwarfs, neutron stars, and black holes, and be able identify the types of stars that become them

Reach for the Stars is a GK-12 program supported by the National Science Foundation under grant DGE-0948017. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

- Students will be able to conceptually describe Einstein's general relativity, and how it gives us the force of gravity
- Students will be able to demonstrate knowledge of gravitational waves, how they come out of general relativity, and some of the data analysis techniques that lead to their discovery

Standards Addressed

Science and Engineering Practices: Developing and Using Models, Using Mathematics and Computational Thinking

Disciplinary Core Ideas: Earth's Place in the Universe, Earth and Human Activity, Matter and Its Interactions, Energy, Waves,

Performance Expectations:

- HS-ESSI-1, Develop a model based on evidence to illustrate the life span of the sun and the role of nuclear fusion in the sun's core to release energy in the form of radiation.
- HS-ESSI-3, Communicate scientific ideas about the way stars, over their life cycle, produce elements.

Time

Approximately 5 30-minute class periods. Recommended breakdown is:

1) Lives of stars, hydrostatic equilibrium, nuclear fusion
2) Stellar deaths and remnants – supernovae and compact objects
3) Intro to general relativity
4) Gravitational waves – theory and discovery
5) Gravitational wave detectors and searches for gravitational waves

Level

High School (grades 9-12)

Materials and Tools

- **Demo Materials:** Large balloons, aluminum foil, pins/thumb tacks, basketball, tennis balls, "Spacetime Table" (or 5' by 5' sheet of spandex), heavy metal ball bearings (~3" in diameter), ping pong balls
- Supernova and Stellar Remnants worksheet (download or see at the end)
- Supplementary python script random walk
- Supplementary python script Nbody simulator

Preparation

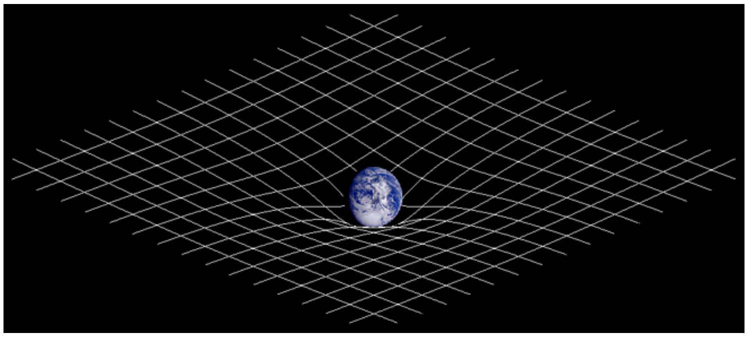
Each demo should be set up before the beginning of class. Supplementary lectures to the presentation should be made. If students do not have computers/notebooks, worksheets should be printed out. Supplementary notebooks can be run on the projector for the class to see. If students have laptops, jupyter notebooks can be circulated through Collaboratory (https://colab.research.google.com/).

Prerequisites

None, but some knowledge of how to *run* (not code) a Jupyter notebook is recommended.

Background

- Stars are the backbone of astronomy, playing a huge role in everything from planet formation to galaxies to the evolution of the Universe as a whole. The evolution of stars is a complex topic, however understanding qualitatively why stars live and die can be portrayed in an approachable and digestible manner which requires no prior knowledge of astrophysics. Stars exist in a balance called *hydrostatic equilibrium*, where the inward pressure from the force of gravity is counterbalanced by the outward radiation pressure from photons that are created during the fusion reactions in the stellar core. This balance keeps the star stable until it runs out of fuel in its core.
- Throughout their evolution, stars are in this perpetual balancing act. Gravity is an unwavering, incessant force always trying to compress the giant ball of gas as much as it can. However, the dense cores of stars attain the right conditions to trigger and sustain nuclear fusion, through which radiation is released and pushes back against gravity. This *radiation pressure* and gravity balance each other out throughout the star's life, keeping it a relatively constant size. However, once a star runs out of fuel, gravity wins the battle and its core begins to collapse. The cores of high-mass stars can form a collapsed core called a *neutron star* held up by an atomic effect known as *degeneracy pressure*. Once the neutron star forms, the envelope of the star will rebound off this core, mostly energized by the neutrinos that are formed from the formation of the neutron star and leads to a *supernova explosion*. Higher-mass stars have so much mass that degeneracy pressure cannot even hold up the collapsing core and will collapse all the way down to a black hole. With nothing left to push back against gravity, the dense core of the star can potentially continue to collapse and form a black hole - an object so compacted with such strong gravity that not even light can escape its gravitational pull if it gets too close.
- Black holes are some of the most exciting and interesting objects in the cosmos, and their exotic nature entices scientific inquiry, as well as a solidified place in science fiction. But how do astronomers know these objects exists in the first place? How do they form? And why are they important in our understanding of the Universe? This lesson plan takes students through the lives and deaths of stars, touches on the modern theory of gravity through Einstein's Theory of General Relativity and how this theory predicts the existence of black holes, and how scientists have recently observed black holes for the first time using an elusive phenomenon known as gravitational waves.

- 
- This begs the question - how can scientists *see* an object that doesn't emit any light? In 1915, Albert Einstein developed a new theory of gravity known as *general relativity*. This theory describes space not as a static, fixed background on which events occur, but a fabric that bends and distorts in the presence of matter. In essence, the more massive and more compact an object is, the more it warps space. Our movement through this 'curved' space is what we construe as the force of gravity.
- A byproduct of this theory is that objects accelerating through this malleable fabric of space cause ripples in space itself, similar to if one was to drag their hand through water. These ripples, known as *gravitational waves*, provide a mechanism for directly detecting black holes - compact objects moving very fast create larger ripples in space. However, even black holes accelerating through space create unbelievably miniscule ripples, and extremely sensitive instruments and sophisticated data analysis techniques are necessary to detect these signals. In September 2015, a century after Einstein's prediction of gravitational waves, this elusive phenomena was finally detected, originating from the collision of two black holes a billion lightyears away.

Teaching Notes

1. The first part of the lesson could revolve around hydrostatic equilibrium - the balancing act that stars perform during the majority of their life. Gravity tries to compress the star into as small of a size as possible, however radiation pressure from the fusion in the star's core pushes back against gravity. To demonstrate this, wrap a balloon with aluminum foil. Pressing on this balloon represents the force of gravity, and the air pressure pushing back against one's hands represents the radiation pressure. Then, one could use a needle to pop the balloon. Now, the aluminum foil can become much more compressed, as the force of gravity (the hands compressing the aluminum foil) has no radiation pressure (air pressure) to maintain the balance. Some objects in space, such as black holes, have no forces that can prevent a gravitational collapse into an infinitesimally small point.
2. The second part of the lesson will be about Einstein's gravity. To demonstrate this theory, have a sheet of spandex (either built into a table or stretched between two desks) and place balls of different densities into it. One will see how the spandex (space) 'curves' with the presence of the balls (matter). Furthermore, the trajectory of other balls rolling through this curved space is not a straight line - the curvature of the space is what gives us our force of gravity!
3. The last part of the lesson is related to the data analysis techniques used to detect gravitational waves. Because the signals are so miniscule, a technique known as 'matched filtering' is used, where a catalog of simulated signals are compared against the data to see if any real signals are present. Students are given many 'template' signals to match against 'data', sliding these templates across the data until a match is found. Finding the simulated signal that best matches the data allows one to know the

Reach for the Stars

properties of the system that generated the gravitational waves, such as how massive the black holes are, how fast they are spinning, and how far away they are.

Assessment

Teachers can give a 5-minute self-evaluation to their students as an assignment after each class.

Understanding the Awesomeness of the Universe
(with simple math)

Background Information

Now that you have learned about how stars live and die, we can start to get a better conceptual understanding supernova and the compact objects that stars can turn in to. Below are some quick calculations that require nothing more than simple algebra, which can help you grasp the energetics of a supernova and the exotic nature of compact stellar remnants such as neutron stars and black holes.

Supernovae

1. A typical supernova releases 10^{44} Joules of energy. How long would the Sun need to burn to match the energy output of such a supernova, given that it release 4×10^{26} Joules per second?

2. The Crab Nebula, a supernova remnant in the constellation Tauris, was observed in 1054 AD as a visiting "star" that for a period of time became the brightest star in the night sky, so bright that historians say one could even see it during the day and read by it at night. The crab nebula has now expanded to a radius of 5.5 lightyears. On average, how fast did the supernova ejecta expand? How does this compare to the velocity of a speeding bullet (~1 km/s)?

Neutron Stars

3. A typical neutron star has about the mass of the Sun (2×10^{30} kg) compacted into the size of Chicago (10,000 m). What is the density of these objects? How does this compare to the density of lead (~11,300 kg/m^3)?

4. Given this density, how much would a tablespoon of neutron star weigh on Earth? A tablespoon is about 15 cubic centimeters. Is this comparable to a car (2000 kg), an elephant (7000 kg), a blue whale (180,000 kg), an airplane (300,000 kg), or a mountain (3 trillion kilograms)?

5. About 500,000 liters (aka 500,000 kilograms) of water go off the Niagara falls every second, dropping about 57 meters to the Niagara river below. We can calculate the gravitational energy of this falling water by the formula $E = m \times g \times h$, where m is the mass, g is the gravitational acceleration (9.8 m/s^2 on Earth), and h is the height from which the water falls. How much gravitational energy does the Niagara falls produce per second? Now, let's pretend Niagara falls was on a neutron star. You can get the gravitational acceleration on the surface of a neutron star just like how we calculate it on Earth: $g = G \times M/R^2$, where G is the gravitational constant (6.67×10^{-11} m^3 kg^{-1} s^{-2}), M is mass of the neutron star (2×10^{30} kg), and R is the radius of the neutron star (~10 km). How much energy do these extreme falls produce per second? How does this compare to the average energy usage of all the humans on our entire planet ($\sim 5 \times 10^{20}$ Joules/second)?

Some neutron stars can spin around at breakneck speeds — up to 1000 rotations per second! Given the size of a typical neutron star (~10 km), how fast would a person standing on the surface of a neutron star be moving? How does this compare to the speed of light (3×10^8 m/s)?

Black Holes

1. One can calculate the radius of the event horizon of a black hole of mass M (i.e. the surface at which light can no longer escape) by using the escape velocity formula: $v_{esc} = \sqrt{2 \times G \times M/R}$ and setting v_{esc} to the speed of light ($c = 3 \times 10^8$ m/s), and solving for R. First, replace v_{esc} for c and write an equation for R, which we call the "Schwarzschild Radius" of a black hole. Given this, how "big" is a black hole that is the mass of the Sun (2×10^{30} kg)? How about Sagittarius A*, the supermassive black hole at the center of the Milky Way, which is 4 million times the mass of our Sun? How about the mast massive supermassive black holes ever observed in the universe (~4 billion times the mass of the Sun)?

2. As you saw in the previous problem, the radius of a black hole is linearly proportional to its mass, meaning if you double the mass M you double the radius R. Oddly enough, this means that as black holes become more and more massive, they become less and less dense (since the volume of a sphere scales as R^3). How massive would a black hole need to be for it to be less dense than water, which has a density of 1000 kg/m^3?