



# Simulate the Sun-Earth-Jupiter system

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## Purpose

The gravitational problem of three bodies in its traditional sense dates in substance from 1687, when [Isaac Newton](#) published his "Principia". Newton took the first steps in the definition and study of the problem of the movements of three massive bodies subject to their mutually perturbing gravitational attractions. Newton also took the first steps in applying his results to the [lunar theory](#), the motion of the Moon under the gravitational influence of the Earth and the Sun. In 1887, Bruns and Poincaré showed that there is no general analytical closed solution for this problem. In fact, the motion of the three bodies is generally non-repeating! That's pretty amazing.

This activity introduces students to the basic dynamics of the three-body problem. The goal is to determine the trajectories of three point-mass bodies in space under the influence of their mutual gravity forces. For the purpose of this activity, we choose the motion of the Sun, Earth and Jupiter, taken together. Students interact with the properties (input) and orbits (output) of the three-body system via an IPython Notebook. IPython notebooks (also known as Jupyter notebooks), allow one to create a document of executable Python code interlaced with HTML. The HTML can be used to embed text, images, and videos along with any python code. Here the interactivity of an IPython notebook is used to engage the students to the complexity of the three-body problem. This notebook implements a numerical simulation of the [three-body problem](#). This is done using the [Julia Language](#) and the [Sundials](#) library in the web-interface of [JuliaBox](#).

## Overview

1. An introduction to the three-body problem. (10 min)
2. An introduction to IPython notebooks and the web-interface of JuliaBox. This will also include a brief overview of the notebook the students will be using. (10 min).
3. The students take time to open up their computers, download the files needed, sign in to JuliaBox and open the notebook in the web-interface. This requires setting up a Google account in case the students don't already have one as well as read some brief instructions on how to proceed step by step (time will vary depending on the previous experience of the students).
4. Students work through the IPython notebook.

The goal here is to play with the physical parameters of the Sun-Earth-Jupiter system and by adjusting them in different values than the ones measured, identify the role of some of them on the outcome orbits



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of the three bodies in the system (40 min). Students will do this following what is written in the notebook and especially focusing on the instructions in the “Simulate the Sun-Earth-Jupiter system” section in the notebook.

a. Students set the masses, eccentricities and semi-major axis of Sun, Earth and Jupiter in the middle of the notebook. Initially they use the measured values for these parameters. Running the notebook they observe the outcome orbits of the three bodies at the end parts of the notebook and compare it with the knowledge they have about our solar system.

b. Students then study three extreme cases for Jupiter to identify the effect of three parameters on the outcome orbits of the three bodies. These parameters are and studies as follows:

i) Students study the effect of eccentricity by simulating a very eccentric Jupiter.

ii) Students study the effect of Sun-Jupiter distance by simulating a very close circular Jupiter.

iii) Students study the effect of Jupiter mass by simulating an Earth with two Suns!

Students are asked to identify the effect of each of these parameters observing what they see in the plots of the bodies trajectories.

In part i) they observe an ejection of a body (here the Earth) due to three-body interactions and orbital-crossing. Jupiter ejects Earth if very eccentric!

In part ii) they observe an unstable system of three bodies and its chaotic evolution.

In part iii) they observe an ejection of a body (here the Earth) by a binary consisting of two Suns. Earth would be ejected if it had two Suns!

c. The lesson is closed with discussions about their results.

If the students finish early, there are optional python challenges that students can attempt. One of them is adjusting and exploring the effect of Earth’s eccentricity.

## **Student Outcomes**

Students will be able to learn the fundamental dynamics of the three-body problem.

Students will be introduced to the complex and chaotic dynamics of the three-body problem.

Students will be able to understand the basic effect of the initial conditions on the three-body trajectories as well as the fate of the system.

Students will have a better understanding of our solar system and its stability.

## **Standards Addressed**

### **HSESS14**

**Use mathematical or computational representations to predict the motion of orbiting**

objects in the solar system.

### **MS-PS2-4 Motion and Stability: Forces and Interactions**

Construct and present arguments using evidence to support the claim that gravitational interactions are attractive and depend on the masses of interacting objects.

### **K-PS2-1 Motion and Stability: Forces and Interactions**

Plan and conduct an investigation to compare the effects of different strengths or different directions of pushes and pulls on the motion of an object.

### **MS-PS2-2 Motion and Stability: Forces and Interactions**

Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object.

## **Time**

This lesson can be completed in a 45-minute class period if everything is set up on the computers ahead of time and if the students have worked with IPython notebooks before. Ideally, the lesson should take two class periods. One period to introduce everything and set everything up, and the second period to do the activity.

## **Level**

This lesson is designed for an advanced Physics course.

## **Materials and Tools**

Technology including software and files:

A python distribution including matplotlib, numpy, scipy, and IPython. In addition, IPython should be updated to at least version 3.1. The Anaconda or Canopy distribution is a good choice as they are comprehensive and free for academics. Find out more [here](#). The folder containing the IPython notebook as well as the associated instructions can be found on my personal google site for Python lessons: <https://sites.google.com/site/fanidosopoulou/>

Equipment needed:

- 1) -A Computer lab or Laptops for the students or groups of students.
- 2) -A projector in the classroom.

## **Preparation**

The teacher should be familiar with the IPython notebook and make sure that it runs on all of the computers.

## **Prerequisites**

No prerequisites, although previous IPython notebook or python experience in the classroom helps.

## **Background**

All relevant background is given in the Introduction in the notebook in detail.

## **Teaching Notes**

Introduction: Give a brief introduction on the three-body problem following the first part of the notebook.

The Activity: The IPython notebook for download as well as instructions on how to login/set up an account in JuliaBox and upload the notebook in the JuliaBox web-interface can be found in my personal website under the subpage 3-body problem in Python Lessons:

<https://sites.google.com/site/fanidosopoulou/> .

The notebook will open up in a web browser. If the notebook opens up in Internet Explorer, copy and paste the url into either Firefox or Chrome. To run the notebook students have to select “Run all” from the “Cell” menu in the toolbar. The notebook says this in the directions, but it may be best to get all of the students to do it once as a class.

If needed the students to save their final plots for each of the exercises. They can do this by checking the Save to File checkbox above the sliders and inputting a file name.

## **Assessment**

Students will answer questions in the notebook.

Class discussion to gauge overall understanding.

## **Additional Information**

N/A