“What’s that over there?”: Retrieving Spatial Information from a Digital Photograph

Peter Ashton and Dianne Lebryk

Purpose

Digital images have become nearly ubiquitous thanks to the availability of cheap cameras, data storage, and high-speed internet. In the modern world, these images are often pervasive to the point that the relationship between the viewer, the camera, and the object photographed goes unexamined. What information does a digital image contain? What information does it not contain? And how can we use such an image to learn about the physical world from a distance?

Overview

Students will examine a series of digital photographs and use the images’ pixels to quantify the 2D space of the image. Combining this information with 2D spatial information from a map, students are able to build a full 3D conception of the object(s) imaged. A final investigation into an observed natural phenomenon challenges the initial geometric assumptions that went into forming the 3D model.

Student Outcomes

Students will be able to:

- Use pixels to describe lengths and locations of features in an image.
- Describe the differences and relationships between image size, angular size, and physical size.
- Combine geospatial information with image analysis to characterize objects in 3D space.

Standards Addressed

- HS-PS4-2: *Evaluate questions about the advantages of using a digital transmission and storage of information.*
- HS-PS4-3: *Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described either by a wave model or a particle model, and that for some situations one model is more useful than the other.*
- HS-PS4-5: *Communicate technical information about how some technological devices use the principles of wave behavior and wave interactions with matter to transmit and capture information and energy.*

Time

Approximately two 45-minute class periods.

Level

Grades 10-12 Environmental Science. Also appropriate for high school Physics or Geometry.

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Materials and Tools

- Computers (recommended 1 per pair of students) with an internet browser installed (Google Chrome recommended)
- Students will need to be logged in with a Google account to access web apps. May be their school-provided account if school uses Gmail, or they can use a personal account or create one for free.
- Handout available here
- Presentation available here

Preparation

Make the necessary technology available to students. Confirm that students have access to the lesson images – they should be placed in a Google Drive and shared so that anyone with the link can view them.

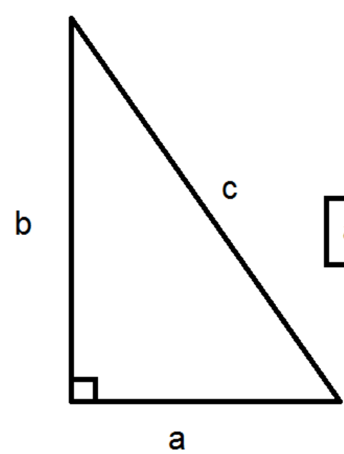
Prerequisites

Students should have some basic geometry experience, working with angles and triangles (though not necessarily trigonometry). Students should be able to find distances between two points given their x- and y-coordinates.

Background

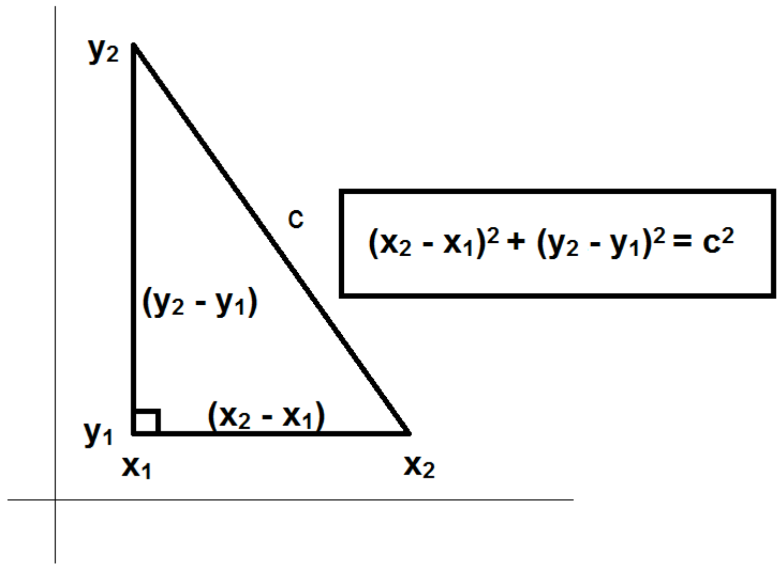
A camera creates an image by taking light rays coming from an object and using a lens to focus them onto something that will be sensitive to brightness and color of the light. In film cameras, the light is focused onto a piece of film, causing a chemical reaction that creates an image. In digital cameras, the light is focused onto a grid of "pixels", each of which electronically records how much light fell on it.

The Pythagorean Theorem tells you the length of the hypotenuse of a right triangle, given the lengths of the other two sides:



$$a^2 + b^2 = c^2$$

Notice that if you wanted to find the distance between two points on a coordinate grid, you could form a right triangle where the lengths of the legs come from the differences in the x and y coordinates:



[Note to teachers: more background is provided within the lesson.]

Teaching Notes

This lesson is intended for students working alone or in pairs with the teacher keeping the class roughly together, though students could also work in small groups to promote more discussion. The attached document is part presentation, part worksheet; ideally the teacher will display it on a projector while students also view it on their individual screens. Questions are embedded throughout. Students should record answers and any calculations on a separate sheet of paper which the teacher may collect.

Part 1 is mainly to give background, though many students may be familiar with the concepts of pixels and resolution. The most important takeaway is that an image provides a coordinate grid that we can use to describe positions, and pixels provide a unit of length with which to measure features in an image.

Part 2 begins to put these ideas into practice. A helpful idea to explain lines of sight is to ask students to imagine extending two pieces of string from their eye to the top and bottom of a building (or whatever they're looking at) and measuring the angle between the two strings. The 35° given as the angular size of the Leaning Tower of Pisa comes from looking up the height of the tower and estimating the distance of the photographer, and computing the angle with an inverse tangent function. Students may take it as given, but some may wonder where the angle came from. The trigonometry is not essential to this lesson. Question 8 is important to gauge understanding. It is easy to measure the size of the tire in pixels via the subtraction method they just used. It is possible to calculate the angular size in degrees because we know the image size and the plate scale (one plate scale describes the entire image). It is *not* possible to find the physical size of the tire, though, as we don't know its distance from the camera. Some students may argue that we can make assumptions based on the height of the person or the typical size of bicycles. Emphasize to them that that is true, but in that case they are adding extra information that's not contained in the original image to draw that conclusion.

Part 3 uses the techniques that students previously developed on the "mystery image". Because students may pick different pairs of landmarks to measure, their image distances may disagree, but all should roughly agree on the plate scale. Students should conclude that the origin of the lights is somewhere around Indiana Harbor in Gary, IN or Whiting, IN around 45 km from Northwestern.

Part 4 asks students: "If the lights are where we think they are, how high off the ground must they be?" They are presented with a schematic diagram of a slice through the center of the Earth with important points labeled. Depending on their math background, they may or may not be familiar with the concepts of a tangent point. This is the place on the curved surface of the Earth where the line of sight just skims the surface at one point. Students should assess their own knowledge – we have just calculated the distance from viewer to light source (EL) and we can look up the radius of the Earth (OA, OT,OB). To continue we must estimate the height of the camera above the surface of the lake. At this point the teacher may switch to the attached presentation on Horizon Geometry. An image of the location where the photograph was taken is provided, and students may make estimates from it. Values of EA between 3 and 10 meters are acceptable, but let students choose their own assumption. One final relationship that's necessary is that the line of sight (EL) is perpendicular to the radius of the earth (TO) forming two right triangles. This is implied by the fact that it is tangent to the surface, but it may need to be given to students depending on their math backgrounds. Some students may ask if we can assume the triangles are similar, or if T is the midpoint of EL – it's not necessarily. With this there is enough information to solve for all the sides of the two right triangles using the Pythagorean Theorem, and the height of the distant building can be obtained. Typical answers are around 100 to 150 meters. Prompt students that that would be a very tall building, and showing the picture of Indiana Harbor, there doesn't seem to be any skyscrapers in the area – did we make an error?

Return to the schematic diagram of the line of sight, and ask what assumptions went into our calculation. Possible answers include:

- the height of the camera – we estimated roughly
- the Earth is a sphere – in fact it's slightly oblate around the equator
- the distance to the light source – we may have mis-measured in the original image
- light travels in a straight line

Students may raise valid objections to the first three assumptions. However, they probably don't affect our answer very much (students may calculate for themselves using a different assumed height, for instance). The last assumption, however, is significantly incorrect in this case. When the lake water is cold, air near the surface is kept colder than the air above it. This effectively changes the index of refraction along the line of sight, bending light rays over the horizon in what is technically called a "superior mirage". Students may be familiar with the inverse effect – hot air above hot pavement is causes light from the sky to bend towards the observer causing the appearance of puddles in an "inferior mirage".

Assessment
Polling students for their results at various stages in the process and comparing different groups' answers will allow outliers to be identified and urged to check their work. Students should roughly agree on e.g. pixel length measurements, calculated plate scale, distance to lights, and heights of buildings.

Additional Information
This lesson was developed for a class where students had internet access via Chromebooks but did not have the ability to download and install any native software. Thus the activities and analyses here are designed to be performed using free browser-based tools. An alternative for classes using PCs or Macs would be measure pixel coordinates using something like MS Paint (coordinates of pointer are displayed below the image) and drawing lines of sight using the ruler tool in Google Earth.