Weighing a Supermassive Black Hole

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Purpose

The discovery of Kepler's Laws have had a tremendous impact on our perspective of the world as a civilization. Typically, students are only exposed to one application of Kepler's Laws (that of the planets orbiting the sun). By extending the topic to a black hole, students will be able to utilize tools they have already learned and apply them to a foreign concept. This will not only allow them to better grasp the scope of Kepler's Laws but also lends an opportunity to learn new material with previously learned material as a basis.

Overview

A supermassive black hole (SMBH) resides at the center of our Milky Way Galaxy, where a cluster of stars known as the S-stars are also found orbiting. Teams of astronomers track the motion of these stars (~27,000 lightyears away) and find their orbits are Keplerian. As a result, using very fundamental concepts, the location and mass of the SMBH can be calculated/determined.

Students will use a NetLogo applet designed to model the motions of the S-stars. By investigating the applet, students can measure the period and semi-major axis of the orbits. Kepler's Laws can then be utilized to determine the mass (and location) of the SMBH.

Student Outcomes

Students will be able to…

- state Kepler's three laws of planetary motion and in particular:
    - know how the first and second law can be used to find a blackhole.
    - know how the third law can be extended to determine the mass of an orbited body.
- define 'proper motion'
- describe where the SMBH in the Galaxy is located and what the S-stars are.
- plot data and determine a linear 'best-fit' line to extract the slope.
- define the inclination of an orbit and explain how the projected (or observed) orbit can differ as a result.

Standards Addressed

(http://www.nextgenscience.org/search-standards-dci)

- **HS-ESS1-4** Use mathematical or computational representations to predict the motions of orbiting objects in the solar system.
- **HS-ETS1-2** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
- **HS-ETS1-3** Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
- **HS-ETS1-4**. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

Time

Two 45 minute class periods. The lesson is flexible enough to extend this estimate to as much as a week long unit or one class period (see teaching notes).

Level

Junior/Senior Astronomy Course

Materials and Tools

- NetLogo software
- NetLogo applet (*both blackhole.nlogo AND coord.png*)
- Computers
- Worksheet
- Ruler
- Visual slides for introduction

Reach for the Stars is a GK-12 program supported by the National Science Foundation under grant DGE-0948017. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

Preparation

Instructors should ensure the NetLogo software (https://ccl.northwestern.edu/netlogo/) is installed on the computers. In addition, the supplied NetLogo applet should be readily available for the students to access on each computer.

Prerequisites

The lesson assumes the students have already been exposed to Kepler's Laws (but can be extended to include a more complete introduction).

Background

Recall that Kepler's Laws of planetary motion describe the orbits of the planets (including earth) around the sun. However, Kepler's Laws are not limited to only our solar system. Anytime there is an object much more massive relative to the orbiting bodies, we can apply Kepler's Laws. The most obvious extension is exoplanets orbiting other stars. Another is a group of stars (the S-stars) 27,000 lightyears away that are observed to orbit a mysterious object at the center of the Galaxy.

Even though the stars are extremely far away, they are moving at such a high velocity that astronomers can track their motions by noting the apparent change in the stars relative location at different observing epochs. This type of motion in the sky is known as *proper motion* and has led to the conclusion the S-star's orbits obey Kepler's Laws.

As a result, the first law can be used to determine the location of the object even though it is not necessarily visible. And finally, by measuring the time for the stars to complete an orbit and the semimajor axis one can use Kepler's third law to calculate the mass of the object. Astronomers have found that the mass is so large and in such a confined region that it must be a SMBH.

Teaching Notes

The students should be introduced to the lesson with a short class discussion of what Kepler's Laws are and why they are important. This should be followed with a 'tour' of the Galaxy to illustrate where the sun (and us) are in relation to the Galactic center. A number of animations can be shown, displaying the S-stars orbiting the SMBH that can be found online. Depending on the level of the class, it may be necessary to expand Kepler's third law to reveal its dependency on the mass of the orbited object. The above should most likely be presented as a slide presentation and techniques such as a 'Think, Pair, Share' can be used to encourage class participation.

After the initial introduction of the topic, a demo of the NetLogo applet should be shown before the students begin working on the supplied worksheet.

A slightly more advanced topic that addresses the inclinations of the orbits is optional and can be omitted if time is constrained.

Assessment

It is suggested to actively engage the students while they work on the worksheet to gauge their progress and understanding. In addition, the worksheet itself can be used as a post-assessment and appropriately graded as the instructor sees fit.

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Name______________________________________ Period______ Date______________

Weighing a Supermassive Black Hole

We will investigate how astronomers measure the mass of the black hole at the center of the Milky Way Galaxy using Kepler's laws. Read all instructions and questions carefully, as each has multiple parts.

NetLogo Applet

Begin the NetLogo applet by double-clicking on 'blackhole.nlogo'. The applet allows you to monitor the motion of stars in orbit around an unknown object. Before beginning, click the 'SETUP' button and then click 'PLAY/PAUSE' when you are ready. Note that the year (beginning at 2015) will progress as the simulation runs.

Question 1

After letting a few of the stars complete their orbits can you determine where the black hole is located? Use the coordinate system to estimate the right ascension and declination of the black hole. Which of Kepler's laws did you use? Describe this law.

Question 2

In your own words, give a quick description of what the eccentricity of an orbit is. Which color star appears to have the most eccentric orbit? Which has the least eccentric orbit?

Question 3

For any particular star, does the speed appear to change throughout the orbit? In what way? Which of Kepler's laws address this? Summarize this law. Is this effect more noticeable for stars with highly eccentric orbits or less eccentric orbits?

Kepler's Third Law

For planetary motion, Kepler's Third Law says that *the square of the orbital period* (P) *is proportional to the cube of the semimajor axis* (a). If we agree to measure the orbital period in **years** and the semimajor axis in **AU,** this equation looks like:

$$a^3 = M \times P^2 .$$

Where M is the mass of the object being orbited (the sun in the case of our solar system). Before utilizing this equation to determine the mass of the black hole, we must first check what the consequences are of using our chosen units (**years** and **AU**) by testing it on our own solar system.

Question 4

Complete the table below and plot your results on the space to the right of the table (as P^2 vs a^3). Remember to label your plot. Finally, using a ruler - draw a best fit line to the data you plotted.

Planet	Period P (years)	Semimajor Axis a (AU)	P^2	a^3
Mercury	0.2408	0.3871		
Venus	0.6152	0.7233		
Mars	1.881	1.524		
Jupiter	11.86	5.204		
Saturn	29.46	9.582		

Question 5

What is the slope of your best fit line? How does this relate to the mass (M) in Kepler's Third Law as written above? It turns out that by using **years** and **AU** as our units, the mass (M) is in units of **solar masses** ($M_{\odot}$). With this knowledge, what is the mass of the sun in solar masses? Does this make sense?

S-Stars

Question 6

Use a ruler to determine the semimajor axis of the five stars orbiting the black hole in the NetLogo applet. In addition, record the period of the orbit. Tabulate the data in the table below.

Star (color)	Period P (years)	Semimajor Axis a (AU)	P^2	a^3

Now, as you did in *question 6* plot the calculated values of P^2 and a^3 in the space provided below. And then draw a best-fit line.

Question 7

As in *Question 5,* determine the slope of your best fit line (should be very different). Now recalling how the slope is related to the mass, calculate the mass of the object the S-stars are orbiting (this is the mass of the supermassive black hole!). Be sure to write what units the mass is in (as determined in *Question 6*). How does this mass compare to that of the sun?

Question 8

In general, Kepler's Laws can be utilized in this fashion if the orbiting bodies are much smaller in mass than the central object. For the solar system, this is applicable since the sun is 330,000 times more massive than the earth. If a typical S-star is ~15 $M_\odot$, how much more massive is the black hole than a typical S-star? Is our application of Kepler's Third Law OK?

Inclination

To a very good approximation, all of the planets in our solar system orbit around the sun on the same plane. Unfortunately for Galactic center astronomers, this is not the case for the S-stars orbiting around the black hole. The result of this, is that the orbits we see the stars trace on the sky are actually '*projections*' of their actual orbit on the sky.

The *inclination* of an orbit is a number that tells us how different the observed orbit is from the actual orbit. In the previous questions, we conveniently took measurements as if there was no inclination. Now, we will 'turn on' the inclination of the orbits to observe how it affects our data and resultant black hole mass.

Remember to hit the 'SETUP' button in the NetLogo applet after turning on the inclination switch.

Question 10

With the inclination turned on, describe how the orbits appear to have changed. Would it be more difficult to determine the location of the black hole in this case? Why or why not?

Question 11

Repeat *Question 6 AND Question 7* with the inclination switch in the NetLogo applet turned on. Create both your table and plot from scratch in the space below.

Question 12

By comparing tables, how did the period and semimajor axes change? Do the changes or non-changes of each parameter make sense? Explain.

Question 13

How did the calculated black hole mass change? Should astronomers take inclination seriously if they want to measure an accurate value?