A Computational Model for Motion With Non-constant Acceleration

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Purpose

The purpose of this lab is to explore motion of a physical system subject to changing accelerations with the aid of computational methods. High school physics curricula typically cover motion under constant accelerations. This represents a very small subset of systems of interest in mechanics. This lesson is designed to introduce computational techniques that allow students to model the consequence of changing accelerations by combining their knowledge of constant acceleration with a computational algorithm.

Overview

This lesson involves recording laboratory data of a pendulum's motion and employing a computational model to simulate the motion that was studied in the lab. Students will take measurements of the pendulum period as a function of starting angle using photo-gate sensors. The students will then edit code that has been partially completed in order to implement an algorithm for approximating an object's motion under generic position dependent forces. The students will first edit the code to simulate the motion of two familiar cases of motion, 0 and constant acceleration, and then edit the code to recreate the data they took in the lab by modeling the motion of a pendulum.

Student Outcomes

Understand how computers can be used to simulate motion of systems that experience accelerations that depend on their position.

SWBAT distinguish between physical systems in which accelerations are constant (or zero) and systems where accelerations are changing.

Collect and display data using digital lab equipment corresponding software (i.e., using the photo-gates).

Students will be able to make predictive plots of position and velocity versus time for objects experiencing constant or zero acceleration.

SWBAT edit pre-written computer code to change the behavior of a program that simulates motion under different forms of acceleration (zero, constant, and variable)

SWBAT explain the correspondence of the sign (i.e. positive/negative) of a velocity or acceleration with its direction and understand the consequence of different combinations of signs of velocity and acceleration on motion.

Standards Addressed

NGSS: HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

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Time

3-4 class periods

Level

Junior Physics

Materials and Tools

- Pendulums
- Photo-gate and software to collect Photo-gate data
- Computers with Net-Logo installed (https://ccl.northwestern.edu/netlogo/)
- Net-Logo Pendulum Model (pendulum.nlogo)
- Worksheet (PendulumNetLogoWorksheet) and lab sheet (Pendulum Lab.pdf)

Preparation

Set up pendulums and ensure that photo-gates can register when the pendulums swing through the gate. Net-Logo will need to be installed on each computer and students will need access to the Net-Logo Pendulum Model.

Prerequisites

Students should be comfortable with basic kinematics. Students should know the equations that describe the evolution of position and velocity with time under zero or constant acceleration and be able to interpret position-vs-time and velocity-vs-time plots. It is good for students to have previously been exposed to NetLogo and be comfortable with editing code in NetLogo.

Background

We have discussed the way objects move when they travel at constant velocity or when they are subject to a constant acceleration. For these types of motion we have been able to derive some mathematical rules that allow us to predict their motion. But the real world is full of types of motion that is clearly much more complicated. We'll be looking at an example of motion under constantly changing acceleration: the motion of a simple pendulum. While it may not be clear how to mathematically account for motion under conditions of continuously changing acceleration, we'll look at how we can make some simplifying approximations and combine what we already know about constant velocity and constant acceleration motion with the powerful capabilities of a computer to do many calculations quickly. We will be able to accurately simulate a pendulums motion using computers without having to solve complicated equations.

Teaching Notes

- The lesson should begin with a review of the types of motion already discussed. The teacher should point out examples of motion from everyday experience that both fall under the category of 'constant acceleration' along with motion that involves changing accelerations.
- The example of the pendulum is introduced as a physical system that experiences a position-dependent acceleration. Briefly discuss the how a pendulum's motion depends on the position in its swing: at the bottom of its swing a pendulum experiences no net acceleration because the force of the string balances the force of gravity while when the pendulum is horizontal in experiences the full acceleration of gravity.

- Students collect time-series data of pendulums. The pendulums should be set up to swing through the photo-gates and students should record the period of the pendulums swing for a few different starting angles. Detailed instructions for students are included in the lab guide.
- The next part of the lesson should take place in the computer lab. Begin by discussing examples of real-world applications of computer modeling. This can include, e.g., models for the motion of the Solar System or stars in a galaxy, modeling molecules and proteins in the context of biology and medicine, modeling building/structures in the context of engineering, etc. Then describe the algorithm that will be used to simulate an objects motion. The pendulum can be abstracted to its position and velocity and thought of as having two behaviors, 'drift' and 'kick', that it enacts for a user-specified amount of time. When the pendulum drifts, it moves along at its current velocity as though it felt no acceleration and the position is updated according to the length of time that it drifts. Expressed mathematically:
    - **DRIFT:**
        - **x_new = x_old + v_old * dt**
        - **v_new = v_old**

  When the pendulum 'kicks', it computes an acceleration based on its current position and then updates its velocity as though it undergoes a constant acceleration. The position of the pendulum is held constant during the kick step. Expressed mathematically:

    KICK:
        - **v_new = v_old + a(x_old) * dt**
        - **x_new = x_old**
- Students should supply code that computes the acceleration based on the position of the pendulum. They should run the simulation for three cases of acceleration: identically 0, constant, and pendulum-like (proportional to -sin(x/l) where x is the pendulum position measured along the arc of its swing and l is the pendulum length). Before running the simulation for each case, students should sketch a predicted position-vs-time and velocity-vs-time graph. When constructing plots for the case of the pendulum, students should consider the motion they observed in the lab.
- After running each simulation, the students should save the position-vs-time and velocity-vs-time graphs that were generated and complete other questions prompted by the worksheet.

The teacher may find it easier to replace the pendulum with a mass attached to a spring as the physical system being modeled. This complicates physical data collection in the lab but it removes the conceptual difficulties of converting the motion of a pendulum to a one-dimensional kinematics problem and/or introducing angular variables to describe the pendulum's motion.

There is some flexibility in how much of the algorithm's code should be written by the students and how much should be supplied by the teacher. Students in advanced classes can write the `drift/kick' algorithm on their own while other classes may only edit code that defines the acceleration the pendulum experiences.

Assessment

Students took formative assessments in the form of pre-lab discussion posts, quizzes on kinematics concepts, and a summative assessment for the unit. The laboratory was graded using a rubric that assessed their ability to distinguish between constant and changing accelerations, interpretation of graphs, and extrapolation of the concept of changing acceleration.