Building Open-Ended Problem Solving Skills:  
The Bus Fleet Project  
Madison Fitzpatrick and Mark Vondracek

Purpose  
This activity is a creative problem-solving project designed to introduce students to the process of tackling problems that are complex and open-ended. Real-world science and engineering problems are very different from classic textbook problems, in which all the necessary information is provided and there is only one correct solution. This project helps students develop the creative problem-solving skills that are important for handling real problems.

Overview  
In this project, students will work in small teams to interpret the problem, identify and gather information, create and execute a plan for solving the problem, and report on their findings.

Student Outcomes  
This is a long project with multiple phases and many learning outcomes involved. Depending on the students’ initial skill levels and the amount of time the class can dedicate to the project, only some of the following learning outcomes may be fully addressed. Ideally, by the end of this project, students will be able to:

1. **Read and interpret problem statements of a complex, open-ended nature**
   - Define the goal or outcome that is desired
   - Define constraints and requirements that must be met
   - Identify information that is necessary to solve the problem

2. **Devise and execute a strategy for solving an open-ended problem**
   - Make assumptions to fill in missing information or simplify the problem
   - Create a mathematical framework and apply appropriate solution methods
   - Support decisions with evidence and logical reasoning

3. **Communicate the solution clearly and persuasively**
   - Write a technical solution paper that describes the solution method and the final recommendation
   - Give an oral presentation of the highlights of the paper

Time  
The time needed to complete this project may vary depending on the skill levels of the students. At a minimum, this project requires six class periods, including one for class presentations, plus additional time outside of class for the students to write the paper and prepare the presentation.
Level
This project was originally done with AP students in physics. It is suitable for any advanced science class, and could be adapted for less advanced high school science classes.

Materials and Tools
- The problem statement (see attached, “Buying Buses”)
- Internet access for research, for at least one class period
- Microsoft Excel, for at least three class periods
- Solver lesson for Excel (see attached, “How to Use Excel Solver”)

Preparation
It is suggested that the teacher try the problem first in order to become familiar with the challenges that the students will face. They will probably struggle a great deal at first, and it is essential that the teacher be able to gently steer them in right direction if necessary. While there is no one correct answer, the teacher should have a sense for what a reasonable answer looks like, so that if a group comes up with something that is very far off, the teacher can notice and help pinpoint the problem.

The teacher should also be familiar with Excel, particularly with the Solver add-on. While not absolutely required, it is a very useful tool for finding a solution to the project problem. If this is not possible, graphical analysis is another good solution approach.

Prerequisites
This project requires students to develop their own equations and inequalities from scratch. While it is not necessary that students have done this before, it is important that they be comfortable and familiar with the representation of quantities and relationships using variables and mathematical symbols. The project also requires an understanding of how to create formulas in Excel.

Background
This lesson does not require any specific content knowledge. It is helpful for students to understand the connection between greenhouse gases and global climate change, so that they can see why a school district may want to minimize its greenhouse gas emissions.

Teaching Notes
It is suggested that this lesson be conducted in the phases described below, so that the students understand what they need to accomplish during each class period. These are not strict requirements, but simply a suggested structure for organizing the tasks involved in completing this project. In addition, it is suggested that each group keep a project notebook (paper or digital) where they record notes about their discussions, research, assumptions, solution method, and results. This will help the students keep track of the decisions they make and the trials and errors that they experience.

1. **Problem definition and brainstorming.** Students form teams and receive the problem statement. The goals of this phase are to define the objective of the problem and its constraints and to identify all the information they need to formulate a solution. Some of the necessary information is provided in the problem statement, but some is not. Students should identify the missing pieces of information.
2. **Gathering information and making assumptions.** In this phase, students should conduct research to find the missing information identified in the previous phase. They may make direct use of information that they find, or they can use various sources (including personal experience) to formulate assumptions that fill in the missing pieces. It is important that students cite the sources that they use and create written justifications for any assumptions they make. Assumptions should be based on reliable information and/or logical reasoning.

3. **Mathematical formulation.** In this phase, students should transform the problem into a set of mathematical statements (equalities, inequalities, and/or expressions). They should develop one statement representing the objective and one statement for each of the constraints. The statements should be constructed using known quantities (such as the information given in the problem statement or assumptions they have made) and variables (namely, how many of each type of bus in the new fleet). This mathematical formulation of the problem is an essential prerequisite for applying a solution method.

4. **Application of solution method.** In this phase, students will use their chosen solution method to find an optimal solution to the problem. It is highly recommended that students use the Excel Solver add-in if possible, but this requires an additional day to teach. If time is available, the materials for teaching this tool are in the attached document, “How to Use Excel Solver.” At a minimum, students will need access to Excel in order to create graphs and test the parameter space to find a solution. Students are likely to struggle with this phase, and they may want to go back and change some of their assumptions or alter their mathematical formula. Because of this, it is helpful to allocate more time to this part of the project.

5. **Interpretation of results.** Once the solution method has provided a result, it is important that they take some time to study it from a logical perspective. Does the result make sense? It is what you expected intuitively? Why or why not? Thinking about these questions is likely to lead some groups to realize that they have made a mistake somewhere. That is fine—that is the purpose of this phase. After thinking about their results, some groups may be ready to start writing their solution paper, but many will probably need to go back and make corrections to their solution process.

6. **Presentation of solution.** The groups should write their papers and prepare their presentations outside of class. However, it will probably be helpful to have a brief in-class discussion of what is expected from scientific writing in general, and from this paper in particular. The attached problem statement (“Buying Buses”) includes some advice on this topic. It is highly recommended that each group be given at least a few minutes to present their solution to the rest of the class and field questions from their classmates. One of the valuable outcomes of this project is that each group will probably take a slightly different approach and get a different solution from the other groups, even though they started with the same information and might have even used the same solution method. It is extremely valuable for students to see this and understand how it came about.

This lesson was originally done with very advanced science students. However, it could easily be modified to be more feasible with students at a different ability level. The following are some suggestions for modification.
• Begin the project with a lesson that introduces the students to the concept of open-ended problem solving. Provide examples of real-world situations in which this skill is important, and show examples of students solving these kinds of problems. Many excellent examples can be found on the websites for the COMAP mathematical modeling competition and Moody’s Mega Math Challenge. Also, explicitly explain the skills students will be learning and practicing, as well as the phases of the project that they will go through.

• Begin each phase of the project with a brief lesson explaining the key tasks and goals for that phase. The purpose is to provide a level of structure and guidance that is commensurate with the students’ ability level.

• To help keep students on track, it may also be useful to break each phase down into specific tasks and guide students through these tasks one at a time.

• For the solution paper, it may be helpful to spend extra time teaching the students how to write scientifically. Holding an in-class editing session, where the teacher can provide individual guidance to each group, is also a good idea.

Assessment
There are three assessment tools that help the teacher evaluate how well the students achieve the learning objectives of this project.

• Informal observations and conversations with students are crucial during the project process. A good way to gauge progress and keep students on track is to float among the groups and observe, periodically asking each group to explain what they have been talking about or working on since you last spoke to them. This method typically starts a lively multi-way conversation, in which the students often reveal questions they have or decisions they are struggling with. Asking them to explain their thought processes out loud to someone outside the group also helps them clarify their own ideas.

• The written solution paper asks students to fully explain their assumptions, mathematical formulations, solution process, and interpretation of the results. This provides ample opportunity for the teacher to evaluate how well the students have achieved the learning objectives.

• The presentation serves mainly to evaluate the third learning objective, which is focused on clear and persuasive communication.