Parabolas and Comet ISON

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1 Purpose

In November 2013, Comet ISON made a close perihelion passage to the Sun, resulting in it's (believed) destruction. Since the orbit of Comet ISON was very nearly parabolic (eccentricity of 1), the data from this provided an excellent real-world example of parabolas. This lesson was designed to reinforce how to plot data and perform a quadratic regression via the student's TI-Nspire calculators, and then how to calculate analytic quantities (the vertex of a parabola and it's distance from a point) from their regression.

2 Overview

During this activity, the students will first hear a quick lecture on the nature of comets and the specifics of Comet ISON. They will then be provided with a worksheet containing position data of Comet ISON's orbit near the Sun. Using their calculators and the given data, the students will calculate the vertex of the parabola, representing the point of closest approach to the sun. The activity ends with a YouTube Video of NASA SOHO images of Comet ISON's passage and destruction by the Sun.

3 Student Outcomes

Students will be able to:

- Enter and plot data in their calculators
- Perform a quadratic regression on this data
- Write down the coefficients of the data in the form

$$y(x) = Ax^2 + Bx + C$$

- Find the vertex of this parabola, corresponding to its perihelion, using the axis of symmetry
- Calculate the distance from the perihelion to the sun using the distance formula

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4 Standards Addressed

The lesson addresses the following Common Core State Standards mathematical standards:

- **CCSS.Math.Content.HSF-IF.A.1** - The function-domain relationship ($y = y(x)$)
- **CCSS.Math.Content.HSF-IF.B.4** - The graph is used to show the vertex of the parabola
- **CCSS.Math.Content.HSF-IF.C.9** - The parabola given in numerical table form is compared to the quadratic regression, and the maximum compared to the analytic vertex.

5 Time

This activity requires approximately 15 to 40 minutes of class time, depending on the class level.

6 Level

This lesson can be used at two levels:

- Algebra II – Lower performing Sophomores and Juniors. Will require more time.
- Algebra II/Trig – Higher-performing Sophomores. Will require less time.

7 Materials and Tools

- TI-NSpire CX Graphing Calculator
- Comet ISON Handout (Attached)
- (Optional) Display with internet access:
  - `https://www.youtube.com/watch?v=6j6nkLnHyG0`

8 Preparation

Apart from printing the handout and ensuring that every student has a calculator, no preparation is required.

9 Prerequisites

This lesson is intended as both a review of the use of the TI-Nspire calculators and of the mathematics of parabolas. It is expected that students will be sufficiently familiar with their calculators to enter data, and only require reminders on how to perform quadratic regressions.

It is also expected that students know how to find the vertex of a parabola via the axis of symmetry and evaluating the function, as well as the distance formula. However, the worksheet has the necessary formulae, so the lesson could be adapted to teach these conpects rather than reinforce them.

10 Background

In late November 2013, Comet ISON made its point of closest approach to our Sun. Since orbits of comets tend to follow parabolas (quadratics functions), we can predict how close the comet got to the Sun. The **perihelion** of an orbit is the point where the comet gets closest to the sun. It turns out that the perihelion is the same as the vertex of a parabola.

In order to complete this assignment, you (the student) will need to remember what a parabola looks like and how to find its vertex. Once you have plotted the data on your calculators, you should be able to fit that data to a parabola using the "quadratic regression" function of your calculators. You can then find the vertex by finding the axis of symmetry, and then writing down what $y(x)$ should be there. Once you have the vertex, you can use the distance formula to find out how close the perihelion of Comet ISON was to the Sun.

11 Teaching Notes

It is generally helpful if the teacher works the class collectively through the questions on the back of the worksheet. Once the introduction has been completed, the students can be left to enter the data and perform a quadratic regression on their own. However, it is useful if the teacher provided hands-on assistance when issues (inevitably) arise with the calculators.

Once the data fitting is completed, the next questions are better answered in a question-and-answer format, by having the students read out their answer after working on each question. This ensures the class remains at the same point in the excersize.

Finally, it is more elucidating to end with the video, as it demonstrates to the students a real-world application of their work in an exciting context.

12 Assessment

Ensuring that the students understand the term **perihelion** and its association with the vertex of a parabola indicates that the students have understood the connection between the math-

ematics and the astrophysics. Otherwise, the correct numerical solution can indicate if the students have performed the exercises correctly.

13 Additional Information

More information (geared towards the general public) can be found at:
`http://solarsystem.nasa.gov/smallworlds/cometison.cfm`.
The data for the worksheet was taken from the educational material at the bottom.

Name:_____ January 8, 2014

Algebra II

1 Comet ISON

In November of 2013, Comet ISON made it's closest approach to the sun. The **perihelion**, or point where the comet is closest to the surface of the sun.

1.1 Asteroid flyby

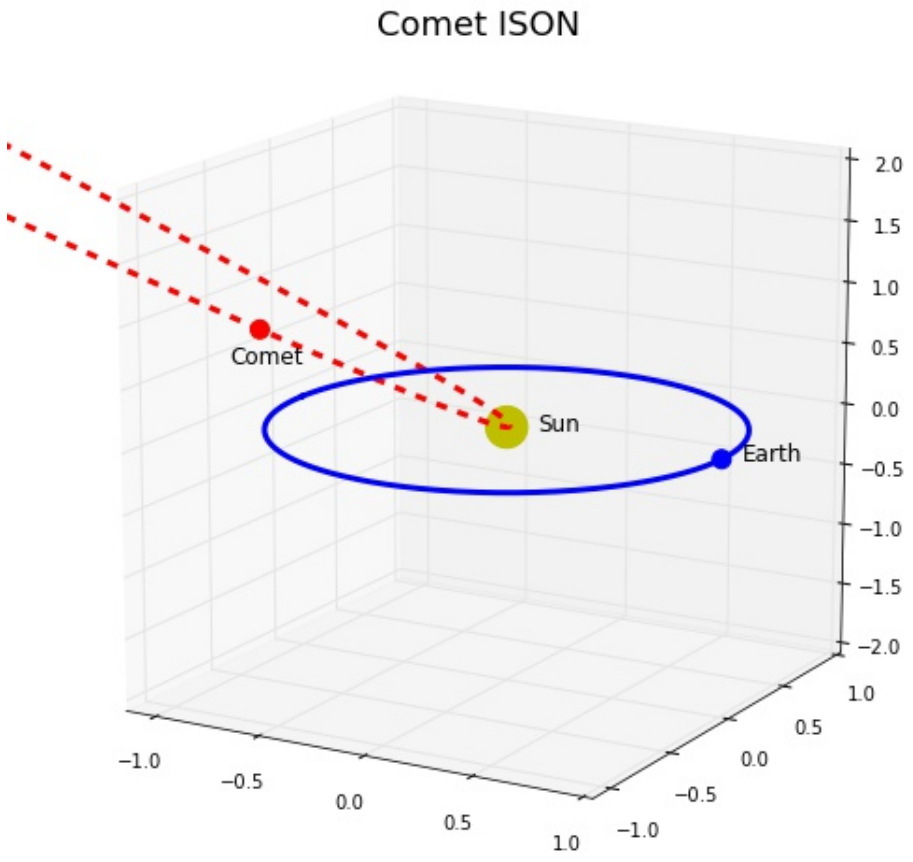

Figure 1: Comet ISON on its way to the Sun.

Table 1: Position data of a comet ISON on an approach to the Sun. This data is courtesy of `solarsystem.nasa.gov`

Date	X (Millions kilometers)	Y (Million kilometers)
November 26, 18:00 UT	-10.5	+0.7
November 27, 13:00 UT	-8.8	+5.6
November 28, 01:00 UT	-7.7	+8.7
November 28, 08:00 UT	-6.3	+10.9
November 28, 14:00 UT	-4.6	+13.7
November 28, 23:00 UT	+3.2	+14.5
November 29, 10:00 UT	+6.6	+9.0
November 29, 19:00 UT	+8.0	+5.6
November 30, 10:00 UT	+9.3	+1.7

1. Using your calculator, fit the data to a parabola
2. Write down the values of the coefficients below:

$$y = A \cdot x^2 + B \cdot x + C$$

- $A =$
- $B =$
- $C =$

3. Find the **perihelion** by doing the following:
   - Find the axis of symmetry. Remember:

$$x = \frac{-B}{2A}$$

   - Find y at the axis of symmetry by plugging it into your parabola

   What are the x and y of perihelion?

4. If the sun is located at x_2 = -0.4, y_2 = 14.7, how far was the **perihelion** of Comet from the sun? Use your answer for the perihelion as x_1 and y_1

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$