Graph Theory and Paths for Food Banks – Joseph Warfel

Purpose

This lesson teaches students the basic elements of graph theory in the context of food bank transportation networks. They use graphs to find and evaluate possible delivery routes. They also learn about algorithms – specifically, algorithms to find and improve paths in graphs. This knowledge will be built upon in succeeding lessons about algorithms to find tours in graphs. The lesson culminates in a writing assignment in which students solve several problems, describe their solution method, and evaluate the quality of their recommended solution in comparison to other possible solutions.

Overview

In this lesson, student study how to make delivery routes for food bank vehicles. The problem is approached through graph theory, a discipline of mathematics used to represent networks. The lesson is broken into four parts which are intended to be taught on separate days (since each part has homework).
In the first part of the lesson, students are introduced to graphs (as defined in graph theory) and are taught the vocabulary necessary to describe a graph. A delivery route is equivalent to a "path" in graph theory. The principal goal of this part of the lesson, other than learning vocabulary, is for the students to calculate the cost of a path (in the food bank context, the distance travelled by the truck making the delivery). This skill is practiced in the homework.
The second part of the lesson focuses on an algorithm for finding a path in a graph. An algorithm is defined as a set of instructions for solving a problem. The students are shown that they actually already know many algorithms by demonstrating the traditional subtraction algorithm. Then, they are shown two other subtraction algorithms, to demonstrate that there are many possible ways of solving a problem. They are given a simple algorithm for finding paths and they apply this to the same food bank networks used in the first period. The homework practices the application of this algorithm.
The third part of the lesson focuses on an algorithm for improving a path in a graph – that is, making it shorter. The improvement algorithm is motivated by a discussion of some of the shortcomings of the path-generation algorithm from the second part of the lesson. The class work practices the path-improvement algorithm. However, in the homework, students attempt to find the shortest paths for four food bank delivery routes and explain how they found their solution. They could use the algorithms from the second and third parts of the lesson, but the problems given have obviously more effective ways to be solved.
The fourth part of the lesson introduces the idea of travel time as an additional concern, by showing that the path with the least distance may require much more time than a longer path. The classwork practices calculating path cost using both time and distance. This period should also include a discussion of how the students solved the problems in the homework – that is, how they improved on the algorithms given to them. The writing assignment asks students to calculate three delivery routes, considering both time and distance in the recommendation they make to the food bank. The writing assignment makes use of a food bank network different from those that the students have already seen.

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Student Outcomes

Learning objectives:

Part 1:

SWBAT understand and use graph theory vocabulary (graph, node, arc, cost, origin, destination, path).

SWBAT calculate the cost of a path.

Part 2:

SWBAT use a path-generation algorithm.

Part 3:

SWBAT use a path-improvement algorithm.

Part 4:

SWBAT calculate the cost of a path in terms of time and distance.

College Readiness Standards:

Throughout:

0.16 BOA 201 – Add and subtract decimals

Part 1:

PSD 303 – Read tables and graphs

PSD 304 – Perform computations on data from tables and graphs

Part 2:

PSD 502 – Manipulate data from tables and graphs

Part 3:

PSD 502 – Manipulate data from tables and graphs

Part 4:

PSD 602 – Interpret and use information from figures, tables, and graphs

Time

The lesson is intended to be completed over four class periods, using about half an hour in each period. The first three lessons include a homework assignment. The last lesson leads to a writing assignment in which students apply what they have learned in the first three lessons to solve a problem on a different network.

Level

9 – Algebra

Materials and Tools

Part 1:

Graph Set 1

Seven paper plates

Tape that is a different color than the floor of the classroom, and which can be written on

Classwork

Homework

Part 2:

Graph Set 1 (though they should have it from before – it is needed for the homework)

Seven paper plates

Tape that is a different color than the floor of the classroom, and which can be written on

Rulers (preferably metric with millimeters or inches with 1/16 increments)

Classwork

Homework

Part 3:

Graph Set 1 (though they should have it from before – it is needed for the homework)

Classwork

Homework

Part 4:

Graph Set 2

Classwork

Writing Assignment

Preparation

Part 1:

Attach paper plates and tape to floor in order to represent the Food Finders Food Bank network. Write the names of the cities on the paper plates and the distances on the tape between the plates. If possible, orient the network so that students at their desks see it as it is printed on their pages (north up).

Part 2:

Make Food Finders Food Bank network on floor as described for Part 1.

Part 3:

No special preparation is needed.

Part 4:

No special preparation is needed.

Prerequisites

This lesson only requires that students can add decimals – everything that they need to know about graph theory and algorithms is included in the lesson. The lesson also makes use of some vocabulary that has a specific mathematical meaning, but for which the common meaning is so similar that confusion should not arise (set, sequence).

The motivation behind finding shortest paths is to reduce travel cost and travel time, so if this would not be apparent to the students, this idea should be discussed before beginning the lesson or during Part 3.

This lesson uses transportation networks from food banks. A discussion of the purpose and operations of food banks could precede this lesson or be included in Part 1; however, it is not strictly necessary in order for students to understand the mathematics of the lesson.

Background

The only background skills necessary to complete this lesson are addition of decimals and the ability to follow directions.

Teaching Notes

(Please refer to the *.pdf key to see classwork and homework in their completed state.)

Part 1:

Introduce the vocabulary by filling in the blanks on the first page of the handout. As each term is introduced, indicate it in the graph below. Then, fill in the blanks to interpret the graph theory ideas in the context of this lesson.

For the three example paths, choose a student to simulate moving along the path by standing on the paper plates and moving to the ones specified in the node sequence. Emphasize the idea that the cost of using an arc is incurred by moving along it, so as the student moves along he or she adds to the cost of the path. Example 3 is the undefined case.

The important point to emphasize in this part of the lesson is correctly calculating the cost of a path. This is not a difficult skill, but it is fundamental to the other parts of the lesson. It is also useful to encourage correct use of vocabulary – i.e., correcting a student who says "dot" instead of "node" or "line" instead of "arc."

Part 2:

Start by reviewing a problem from the homework if they had difficulty on it.
Introduce the definition of an algorithm. Mathematics education is, in large part, about teach students algorithms. Therefore, they know a lot of algorithms although they might not realize it. An example is the subtraction algorithm. Show the algorithm they know on the first problem, then show them some alternate subtraction algorithms on the other problems (see http://www.sonoma.edu/users/w/wilsonst/courses/math_300/groupwork/altsub/default.html) to demonstrate that, for some problems, there are many potential algorithms that produce good answers.
To motivate the path-generation algorithm presented here, remind the students of the short answer problem on the homework. They probably will have said that shorter paths are better for the food bank in order to save time and fuel. Therefore, this algorithm is constructed so that it always takes the shortest path that will bring it closer to its destination. For the two examples, as in the first part of the lesson, have a student move along the arcs to generate the path. When a student is at a node, emphasize the instructions of the algorithm – specifically, to look at all of the arcs, choose the shortest one, use that arc if it brings you closer to the destination, and otherwise look at the next-shortest arc. Give them the last problem in the classwork to do on their own. Then, do several problems from the homework (which uses the larger, more complex graph of the Northern Illinois Food Bank) to ensure that they are able to do the problem without the kinesthetic aid of a classmate walking around the network.
Note: In order to apply the "shortest arc" algorithm, it is sometimes necessary to measure the distance between two nodes with a ruler in order to determine whether the node that lies at the other end of a short arc is closer to the destination than the current node. In the problems given, this could arise in classwork problem 1 and homework problems 2, 5, and 6.
Although the "shortest arc" algorithm is easy to apply, it can provide terrible solutions. This type of algorithm is called "myopic" because it only makes one decision at a time, and after making a decision (choosing an arc), the decision cannot be changed. Students may become frustrated when the algorithm does obviously stupid things. It may help to tell them that, in future parts of the lesson, they will be given the opportunity to create a better algorithm. Also, the next part of the lesson is specifically about an algorithm that will improve a poor solution generated by another algorithm. In some optimization problems, a difficult step is just finding a solution of any quality, so even a terrible solution-generation algorithm is useful.
Another shortcoming of the "shortest arc" algorithm is that it does not always find a solution. This arises in question 6 of the homework.

Part 3:

Start by discussing the short answer question in the homework. The students should have noticed some obvious flaws in the "shortest arc" algorithm: it does not always find a solution, it often chooses an inefficient route when a better route (from a human perspective) clearly exists, and others. The shortcomings of our path-generation algorithm motivate the path-improvement algorithm of this lesson.
The algorithm presented here is called "jump" because it attempts to "jump" over the next node in the path. It is similar to some simple improvement algorithms used for tour-finding problems. This algorithm could be very confusing for students, so go through the examples slowly. The students will discover that "jump" has its own problems – for example, as presented, it can only "jump" one node at a time, the consequences of which are emphasized in question #3.
The last question on the classwork (Do you think you can do better than the "shortest arc" and

“jump” algorithms?) is meant to motivate the homework, in which students find paths in any way that they like. The short answer in the homework could be a rough draft for the second paragraph of the writing assignment. Understanding of the algorithms is not necessary to complete the homework.

Part 4:

The purpose of the fourth part of the lesson is to introduce the other arc cost of relevance in logistics, time. Routing decisions must often balance competing concerns of distance travelled (to reduce fuel and maintenance costs) with time travelled (to reduce labor costs, especially overtime, and in the food bank context to ensure that food is properly stored).

The example in the classwork presents a case in which the shortest path (in distance) is not the shortest path (in time). In the other two problems, this conflict does not arise. This part of the lesson equips the students to answer the questions posed in the writing assignment.

Writing Assignment:

In the writing assignment, the students must find paths for three delivery routes using a graph of the West Texas Food Bank, included in Graph Set 2. A rubric is included that may be given to students as part of the assignment. The shortest routes from Odessa to El Paso and from Odessa to Alpine are straightforward to find. For the route from Odessa to Lamesa, the best responses should include two alternatives: Odessa – Andrews – Lamesa has a cost of 86.2 miles and 1:40, while Odessa – Midland – Lamesa has a cost of 78.7 miles and 1:42.

Assessment

Students will complete some problems in class, so that their mastery can be assessed immediately. Assessment will also take place through homework and the final writing assignment.
Note: Starred (*) problems in homework are meant to be more difficult.

Additional Information

Regarding the food banks:
Food Finders Food Bank serves a predominantly rural area in Indiana. Its headquarters is in Lafayette.
The Northern Illinois Food Bank (NIFB) serves the collar counties around Chicago and several rural counties in the northern part of the state. Several of the nodes in the NIFB graph (Chicago, Northbrook, Rosemont, Schaumburg, Addison, Burr Ridge, Hazel Crest, and Country Club Hills) represent cities that are not served by NIFB, but NIFB trucks often pass through them. NIFB has warehouses in Geneva (the headquarters), Rockford, and Park City.
The West Texas Food Bank serves a vast rural area that covers one-sixth of the state of Texas. It has facilities in Odessa, El Paso, and Alpine. Although food bank trucks could make deliveries through Carlsbad, New Mexico (a node included in the graph), this city is not served by the food bank (since it is not in Texas) and such a route would probably only be used if I-10 were unavailable.

This lesson teaches the graph theory terminology traditionally used in logistics. Mathematicians who specialize in graph theory generally use a slightly different set of terms; specifically, “vertex” instead of “node,” “edge” instead of “arc,” and “weight” instead of “cost.” For some students, it might be interesting to introduce these alternate terms and examine the connections with geometry – for example, polyhedron coloring.
The algorithms introduced in this lesson are not traditional algorithms used in logistics. They were designed to be relatively easy for the students to implement, and they resemble heuristic algorithms used for generating and improving *tours* (not paths) in networks. The shortest path problem is “easy” (that is, it is in P, not NP), and there exists an algorithm to solve it to optimality “quickly” (that is, in polynomial

time), so algorithms of the sort presented here are not necessary for the shortest path problem.
This lesson could be augmented with or preceded by a lesson about the Königsberg Bridge Problem, the puzzle that led to the development of graph theory by Euler in the eighteenth century. Euler's method for solving the problem is not relevant to logistics, and uses vocabulary not developed in this lesson, but is interesting and approachable for high school students.